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ANALYSIS OF THERMAL STRESSES IN THIN CIRCULAR PLATE DUE TO MOVING HEAT SOURCE

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ABSTRACT

It is necessarily important to investigate thermal stresses in a thin circular plate due to internal moving heat source. In the present research article, a problem of thermal stresses in circular plate is examined by integral transform technique. This paper convolutes a plot of thermal stresses in thin circular plate due to internal moving heat source. Transfer of heat takes place from the center of plate to periphery of the plate. The behavior of stresses due to initial temperature and final temperature is investigated from lower face to upper face by keeping circular edge thermally insulated. Transfer of heat takes place by conductive mode from lower to upper surfaces. The plate mentioned in the present article is studied on the assumption that there is no effect of surrounding temperature. As a unique case, metallic plate is considered and the outcomes for thermal stresses are computed numerically and graphically. The obtained results based on gold, silver and copper established that maximum strain has occurred in a round plate of silver followed by gold plate.

KEYWORDS: ***Circular Plate, Moving Heat Source, Thermo Elastic Problem & Thermal Stresses***

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NOMENCLATURE

T	*Temperature of Circular plate (^0C)*
$g(r,\emptyset,z,t)$	*Volumetric energy generation*
K	*Thermal conductivity (W/mK)*
μ	*Lame Constant*
$\Psi(r,z,t)$	*Displacement function*
α	*Thermal Diffusivity (m2/s)*
v	*Poisson's ratio*
a_t	*Coefficient of linear thermal expansion (/K)*

1. INTRODUCTION

The properties of solid material are sensitive to change in temperature. The solid body can compress and expand on account of variation in temperature. The adjustment in temperature measurement is found to rely upon temperature change legitimately. A rejoinder of material to internal heat source produces the stresses on the material. This reaction of material reveals the most important characteristic of solid called as thermo elasticity.

Thermo elasticity of solid spreads over the study of temperature changes, mechanical deformation and thermal energy which is evaluated in terms of stresses occurred in a solid. A detailed literature survey revealed that number of experimental works has been carried out on the proposed topic of the present research article. In the literature, it has been assumed that the upper and lower edges are thermally insulated and entire thermal energy is transferred through the bulk of disk or circular plate, which has been delineated in various research papers as observed in detailed survey. Nowacki W. [1] has assessed steady state thermal stresses in a thick circular plate by means of an axis symmetric temperature distribution on the upper face with zero temperature on the lower face and circular edge. Roy Choudhary [2-3] elaborated on the normal deflection of a thin clamped circular plate subjected to ramp type heating of a concentric circular region of the upper face, which satisfied the time dependent heat conduction equation. P. C. Wankhede [4] elaborates on determination of quasi static thermal stresses observed in circular plate when exposed to arbitrary initial temperature on upper face with lower face at zero temperature. In this, circular edge has been kept thermally insulated. Gokulwar and Deshmukh[5] studied thermal stresses in a circular plate with heat sources. Different authors studied and analyzed the thermal stresses in a thin circular plate. Thermal stresses on account of partially distributed and axially symmetric heat transfer along the outer curved surface by using integral transform technique. The results are found to contain infinite series which holds Bessel's functions and have been illustrated numerically. Deshmukh et. al. [6] emphasized on an inverse quasi static thermal deflection problem for a thin clamped circular plate. A thin clamped circular plate in which, a heat flux is prescribed on an internal cylindrical surface of the plate, solved by an inverse axially symmetric quasi-static problem of thermoelasticity and generalized integral transform technique has been applied to obtain suitable heat exchange conditions on the upper and lower surfaces of the plate. Deshmukh et al. [7] discussed quasi static thermal deflection of a thin clamped circular plate due to heat generation. This paper has given the analysis of stresses in circular plate by keeping upper surface, thermally insulated with zero temperature at lower surface. Gaikwad et al. [8] determined nonhomogeneous heat conduction problem and its thermal deflection due to internal heat generation in a thin hollow circular disk. This article dealt with thin hollow circular disk at unsteady state temperature. It has been calculated with the boundary conditions kept at different temperatures. Gaikwad[9] studied two dimensional steady state temperature distribution of a thin circular plate due to uniform internal energy generation. This paper elaborated on the evaluation of temperature, displacement, and thermal stresses in thin circular plates by means of uniform internal heat generation within it. Finite Hankel transform technique is applied to solve the heat conduction equation. Ishaque Khan et al. [10] determined inverse quasi static unsteady state thermal stresses in a thick circular plate. Authors used Goodier's and Michell's functions to find the displacement components and its associated stresses. The results contained the terms of Bessel's function. By considering special functions and illustrated graphically, results are met for unknown temperature, displacement, and stresses.

The present composition highlighted the evaluation of thermal stresses in a thin circular plate under steady temperature field. A thin circular plate has been studied by keeping outer edge thermally insulated and providing constant initial temperature, and arbitrary heat flux is applied on the upper face with lower face at initial temperature. The exchange of heat occurred through heat transfer at lower boundary surface. The present article contains new and novel contribution of thermal stresses in quasi-static thin plate under steady state. The physical stresses obtained here find its application in engineering problems, particularly in the determination of the state of strain in thin circular plate constituting foundations of containers for hot gases or liquids, in the foundations of furnaces, heating plates.

In present research article, the circular plate has been considered and stresses induced in it are studied in detail. The key points of this article are as below:

- The governing time dependent heat equation with the thermoelastic equation of the thin circular plate under study is formulated as a boundary value problem.
- Integral transform technique is applied to solve the time dependent heat conduction problem.
- Based on the derived temperature distribution, the closed form solutions of the thermal stresses in a thin circular plate are obtained.
- Results are numerically verified and the effects on induced thermal stresses are examined.

1.1. Problem Formulation

Consider two dimensional thin circular plates under unsteady state temperature field of radius r and thickness h occupying space D:$0 \leq r \leq a; 0 \leq z \leq h$ as shown in figure 1. The fixed circular edge (r=a) and the center (r=0) i.e. along the radius is thermally insulated. The plate is monitored with variation of temperature along lower (z=0) and the upper surface (z=h). The plate is also studied under generation of uniform internal energyg_0. The heat source moves on the plate surface along a circular trajectory at radius r'round the center of the plate. Thermal stresses in a circular plate due to uniform internal heat generation are required to be determined.

The temperature distribution $T(r, \emptyset, z, t)$ of the plate is governed by the differential equation of heat conduction [12]:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g_0}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1}$$

Where, k is thermal conductivity and α is thermal diffusivity of the material of the plate, $T(r, \emptyset, z, t)$ is temperature.

With initial and boundary conditions are given by

$$[T(r,z,t)]_{t=0} = F(r,z) \tag{2}$$

$$\left[\frac{\partial T}{\partial r}\right]_{r=0} = 0 \tag{3}$$

$$\left[\frac{\partial T}{\partial r}\right]_{r=\mathrm{a}} = 0 \tag{4}$$

$$\left[\frac{\partial T}{\partial z}\right]_{z=0} = f_1(r,t) \tag{5}$$

$$\left[\frac{\partial T}{\partial z}\right]_{z=\mathrm{h}} = f_2(r,t) \tag{6}$$

Thermal Stresses

The differential equation governing the displacement function $\psi(r,z,t)$ is given by [13]

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} = (1+\nu)a_t T \tag{7}$$

Where $\psi = \frac{d\psi}{dr} = 0\ at r = a$

ν is the poisson's ratio and $a_t$is the coefficient of linear thermal expansion.

The stresses σ_{rr} and $\sigma_{\theta\theta}$ are given by [14]

$$\sigma_{rr} = \frac{-2\mu}{r}\frac{d\psi}{dr} \tag{8}$$

$$\sigma_{\theta\theta} = -2\mu\frac{\partial^2\psi}{\partial r^2} \tag{9}$$

And strain is given by $\epsilon_{rr} = \frac{\partial\psi_r}{\partial r}$ (10)

$$\epsilon_{\theta\theta} = \frac{\psi_r}{r} + \frac{1}{r}\frac{\partial\psi_\theta}{\partial\theta} \tag{11}$$

The surface of the circular plate at $r = a$ is assumed to be traction free. The boundary conditions can be taken as $\sigma_{rr} = 0\ atr = a$

Initially $T = \psi = \sigma_{rr} = \sigma_{\theta\theta} = 0\ a\_tt = 0$

Equations (1)-(11) constitute the formulation of thermo elastic problem.

2. ANALYSIS

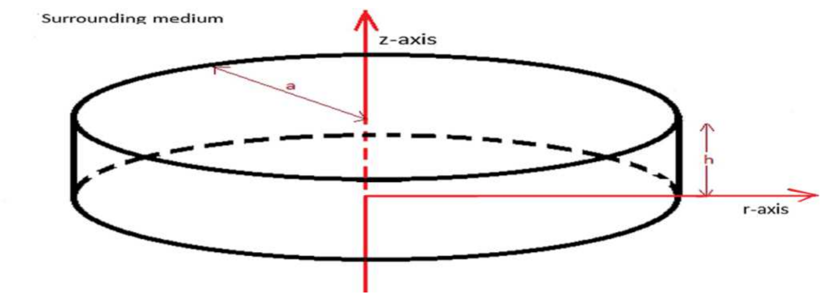

Figure 1: Circular Plate

The solution of the problem in series form is obtained by using integral transform technique. To obtain the expression for the temperature function $T(r,z,t)$, the finite Fourier cosine transform is applied twice [14]

$$\bar{\bar{T}} = \bar{\bar{F}} - \frac{F}{\alpha Q}e^{-\alpha Qt}$$

Applying inverse finite Fourier cosine transform two times:

$$T = \frac{4}{ah}\sum_{m,n=0}^{\infty} F(r,z)(1-e^{-\alpha Qt})\cos\left(\frac{m\pi r}{a}\right)\cos\left(\frac{n\pi z}{h}\right) \tag{12}$$

Special case: $F(r,z,t) = (r-a)(z-h)$

$$T = \frac{4}{ah}\sum_{m,n=0}^{\infty}(r-a)(z-h)(1-e^{-\alpha Qt})\cos\left(\frac{m\pi r}{a}\right)\cos\left(\frac{n\pi z}{h}\right) \tag{13}$$

$$\psi = \frac{4A}{ah}\sum_{m,n=0}^{\infty}(r-a)(z-h)(1-e^{-\alpha Qt})\cos\left(\frac{m\pi r}{a}\right)\cos\left(\frac{n\pi z}{h}\right) \tag{14}$$

$$\sigma_{rr} = \frac{-8\mu(1+\nu)a_t a^2}{ah[(a-r)m^2\pi^2 r+a^2]}\sum(z-h)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\cos\left(\frac{n\pi z}{h}\right)\left[\cos\left(\frac{m\pi r}{a}\right)-(r-a)\frac{m\pi}{a}\sin\left(\frac{m\pi r}{a}\right)\right] \tag{15}$$

$$\sigma_{\theta\theta} = \frac{8\mu A}{ah}\sum\left[2(z-h)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\cos\left(\frac{n\pi z}{h}\right)\frac{m\pi}{a}\sin\left(\frac{m\pi r}{a}\right)+(r-a)(z-h)\frac{m^2\pi^2}{a^2}\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\cos\left(\frac{m\pi r}{a}\right)\cos\left(\frac{n\pi z}{h}\right)\right] \tag{16}$$

$$\epsilon_{rr} = \frac{-8A}{h}\sum(z-h)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\left(\frac{m\pi}{a}\right) sin\left(\frac{m\pi r}{a}\right) cos\left(\frac{n\pi z}{h}\right) -$$

$$\frac{4A}{h}\sum(r-a)(z-h)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\left(\frac{m^2\pi^2}{a^2}\right) cos\left(\frac{m\pi r}{a}\right) cos\left(\frac{n\pi z}{h}\right) \quad (17)$$

$$\epsilon_{zz} = 16A\sum(r-1)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right) cos(\pi r)\frac{\pi}{0.2}\sin\left(\frac{\pi z}{0.2}\right) -$$

$$8A\sum(r-1)(z-0.2)\left(1-\frac{e^{-\alpha Qt}}{\alpha Q}\right)\frac{\pi^2}{(0.2)^2} cos(\pi r)\cos\left(\frac{\pi z}{0.2}\right) \quad (18)$$

Where $A = \frac{(1+v)a_t(r-a)a^2 r}{[(a-r)m^2\pi^2 r + a^2]}$

2.1. Numerical Results and Discussions

Calculation and results for the response of temperature, displacement and stresses are carried out along the radial and axial direction for the fix time and the constant surrounding temperature. Copper plate is chosen for numerical calculations. In the calculation process, the material constants necessary to be known are:

Radius of the circular plate, a=1 Thickness of a circular plate, h=0.2

Temperature of surrounding medium is 25^0 C.

Thermo physical properties of Copper, Silver and Gold metals are as below:

Properties		**Copper**	**Silver**	**Gold**
Thermal diffusivity	a	$1.11X10^{-4}$	$1.6563X10^{-4}$	$1.27X10^{-4}$
Thermal conductivity	K	386	415	327
Poisson Ratio	v	0.36	0.365	0.42
Laméconstant	μ	$47.05X10^6$	$25.27X10^6$	$26.05X10^6$
Coeff. of Thermal Expansion	a_t	$17X10^{-6}$	$18.5X10^{-6}$	$14.0X10^{-6}$

2.2. Graphical Representation

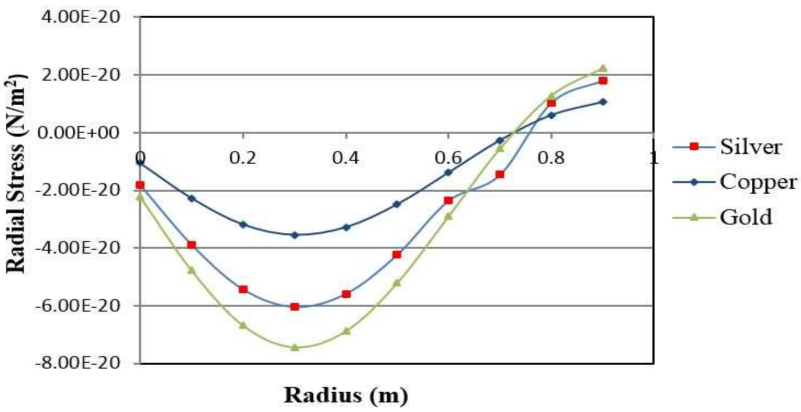

Figure 2: Radius Vs Radial Stress

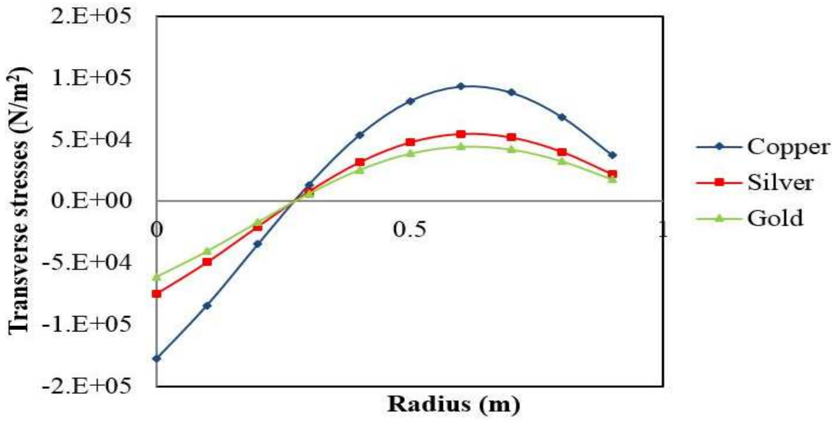

Figure 3: Radius Vs Transverse Stress

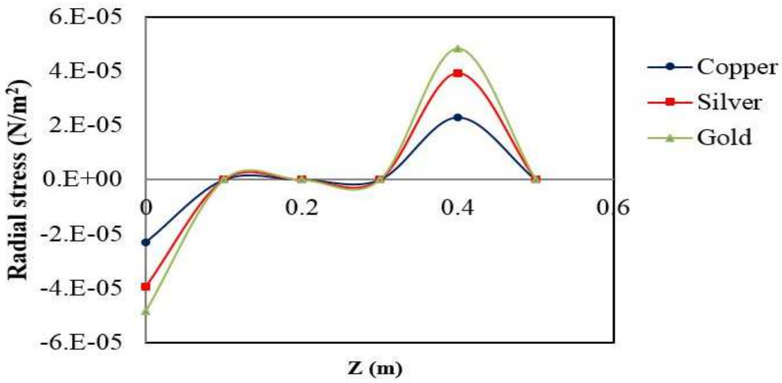

Figure 4: Z Vs Transverse Stress

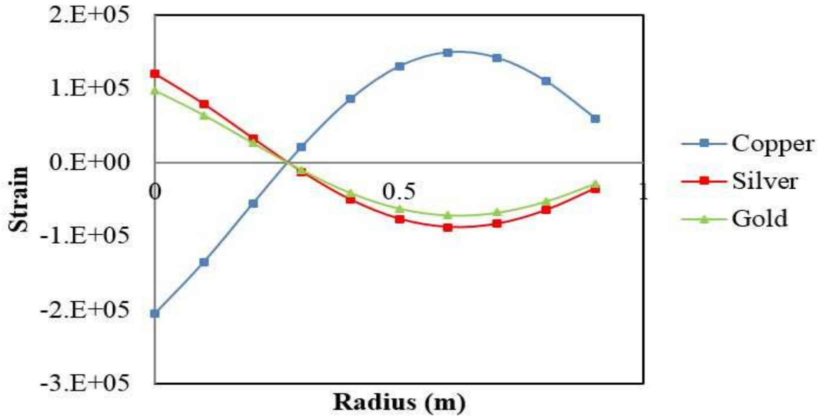

Figure 5: Radius Vs Strain

Figure 2: As depicted in Figure 2, at initial temperature, very small and negligible stresses are found in all these three materials. At the centre of circular plate, stresses begin to transfer towards the edge of the plate. Hence stresses at the core part of the plate remains unchanged. As heat induces through the plate, radial stress is reported to increase and shifted towards the maximum. This maximum stress along the radius continues towards the edge of the plate. The transverse stress observed in circular plate is minimum and stress has been transferred along the radius. The maximum stress variation observed in gold and silver and copper lags behind the gold. Minimum stress observed in copper while maximum observed in gold.

Figure 3: As depicted in Figure 3, the combined effect of stress along the axis and radius is illustrated in Figure 3. Inner core of circular plate and initial thickness along the axis from the centre remains unaffected since negligible stresses are observed. An axial stress behaves in proportion with radial stresses. As radial stress increases, axial stress also increases. But this stress attains its maximum and then it starts moving towards its minimum. This is the saturation point of stress in circular plate. Comparing the transverse stresses in three materials, it is observed to minimum in gold and maximum in copper. It is exactly reverse to stresses along the radius. At the peak of these materials the maximum transverse stress is observed at a particular radius, beyond that stresses decreases gradually.

Figure 4: As portrayed in Figure.4, negative value of stress shows the inner part of circular plate remains unaffected. Hence, this small stresses are neglected. Due to transfer of heat, this stress moves towards the surface of circular plate along the axis. Further, it is observed along the plateau region, the stress remains approximately constant. But after this, the stress changes remarkably. It increases to its maximum for all three materials and these peak values are different for three materials. The stresses observed in gold are more than silver and copper. Copper remains at the lower with small peak value of stress. Hence, it shows the stress handling capacity of three materials. The stress handling capacity of copper is more than gold and silver.

Figure 5: As depicted in Figure 5, the relation of strain produced in silver, gold and copper is illustrated in Figure 5. It has been observed that the strain produced in silver and gold is maximum as compared to copper. The expansion of circular plate of copper is less as compared to gold and silver. Prominent change of strain in silver has been detected. Hence, the strain in silver is more than gold and copper. But, the strain in silver and gold decreases from its maximum and attains its original shape. For copper, initially negligible change is observed and sudden increase in strain is produced. It reaches to its maximum and then starts decreasing to its original.

3. CONCLUSIONS

In the present research article, radial and axial stresses along with strains have been outlined. A circular plate is subjected to heat source from upper surface to lower surface at initial temperature. Initially each metal is found immaterial, for example center of the plate stays unaffected along both hub and range. In combined stresses along axis and radius, the proportional relationship created between the transfers of heat, radial and axial stresses. In present research paper, when three materials viz. Gold, Silver and Copper are analyzed and compared their properties as the ductility and malleability have been verified. Copper is progressively powerful in dealing with the thermal stresses. About Gold and Silver, these are very sensitive to change in temperature due to applied heat source. Here, endeavor is made to look at these three materials based on exchange of heat through the greater part of circular plate. Though copper and silver have smaller stresses as compared to gold, the conduction in gold keeps going longer. Also, the relation of strain in three materials is presented graphically. Maximum strain happened in a round plate of silver and afterward in gold. As compared to gold and silver, copper has small strain produced. Elasticity of volume is established and verified for these three materials.

REFERENCES

1. *W. Nowacki, 1957, The State of Stresses in a Thick Circular Plate due to Temperature Field, Bulletin of Polish Academy of Sciences Series Technical Sciences 5, 227.*
2. *S. K. Roy Choudhary, 1972, A Note of Quasi Static Stress in a Thin Circular Plate due to Transient Temperature Applied Along the Circumference of a Circle Over the Upper Face, Bulletin del'Academie Polonaise des Sciences Serie des Sciences Tech. 20-21.*
3. *S. K. Roy Choudhary, 1973,A Note on Quasi Static Thermal Deflection of a Clamped Circular Plate due to Ramp-type Heating of a Concentric Circular Region of the Upper Face, Journal of Franklin Institute, 206, (3).*
4. *P. C. Wankhede, 1982,On the Quasi Static Thermal Stresses in a Circular Plate, Indian Journal Pure Applied Mathematics, 13(11), pp. 1273-1277.*
5. *V. S. Gokulwar and K. C. Deshmukh,2005,Thermal Stresses in a Circular Plate with Heat Sources, Journal of Indian Academic Mathematic, 27(1), pp. 129-141.*
6. *K. C. Deshmukh and N. L. Khobragade,2005,An Inverse Quasi Static Thermal Deflection Problem for a Thin Clamped Circular Plate, Journal of Applied Science. 28, pp. 353-361.*
7. *Othman, A. A., Osman, M. A., Wahdan, M. H., & ABED-ELRAHIM, A. G. (2014). Thermal annealing and UV induced effects on the structural and optical properties of capping free ZnS nanoparticles synthesized by Co-precipitation method. International Journal of General Engineering and Technology, 3, 9-16.*
8. *K. C. Deshmukh, S. D. Warbhe and V. S. Kulkarni, 2009, Quasi Static Thermal Deflection of a Thin Clamped Circular Plate due to Heat Generation, Journal of Thermal Stresses, 32(9), pp. 877-886.*
9. *K. R. Gaikwad and K. P. Ghadle, 2012, Nonhomogeneous Heat Conduction Problem and its Thermal Deflection due to Internal Heat Generation in a Thin Hollow Circular Disk, Journal Thermal Stresses, 35(6), pp. 485-498.*
10. *K. R. Gaikwad, 2016,Two Dimensional Steady State Temperature Distribution of a Thin Circular Plate due to Uniform Internal Energy Generation, Cogent Mathematics 3(1), pp. 1-10.*
11. *Ishaque Khan, LalsinghKhalsa and Vinod Varghese,2017, Inverse Quasi Static Unsteady State Thermal Stresses in a Thick Circular Plate, Cogent Mathematics, 4(1), pp. 1-10.*
12. *Naotake Noda, Richard B. HetnarskiandYoshinobuTanigawa, 2003,Thermal Stresses, Second Edition, Taylor and Francis, New York, pp259-261.*
13. *M. N. Ozisik, 1968,Boundary Value Problems of Heat Conduction, International Textbook Company,Scranton, Pennsylvania, pp. 148-163.*
14. *Mahmood, I., Jameel, W. W., & Khaleel, L. A. (2013). Improved oxidation resistance for thermal barrier ceramic coating project. Int. J. Res. Eng. & Technol, 1(1).*
15. *N. M. Ozisik, 1968, Boundary Value Problem of Heat Conduction, Dover Publication, INC. Mineola, New York.*
16. *I. N. Sneddon, 1972, The Use of Integral Transform, McGraw Hill, New Yark, pp. 235-238.*
17. *N. Noda, R. B. Hetnarski and Y. Tanigawa, 2002, Thermal Stresses, Second Edition.*